An investigation on the influence on dark matter on the movement of planetary bodies

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Research Question: To what extent does the presence of Dark matter influence the movement of celestial bodies?

Abstract

When investigating the movement of celestial bodies millions of light years away appears rather simple on paper. However, when we decide to consider the plethora of factors that contribute the most, especially with due respect to gravitational attraction and the universal redshift in the universe, altering our observations, it appears that the more we attempt to get closer to an answer by considering more factors, the more questions we end up dwelling on. However, the conceptual theory of dark matter, though its interpretations are of much ambiguity, presents itself as perchance a key towards the explanation of the unexplainable movement of various celestial bodies.

I had decided to undertake this endeavor by investigating the movements of various celestial beings using secondary data from a plethora of sources elaborated more on in the Bibliography. Additionally, I had considered the various perspectives and interpretations of the influence of dark matter towards my final argument that I had concluded on. The data once collected, was processed and modeled integrating python towards establishing a visual representation with direct correlation.

What I had ended up concluding is perchance something that my prior self would considerably disagree with. The investigations reveal that dark matter does contribute significantly towards the movement of planetary bodies, especially those henceforth void of any orbit or in other words, rogue bodies. However, the argument is completely squandered when considering the movement of bodies that orbit any system of planets or stars.

In conclusion to my investigation, the argument of dark matter proving to be a significant contributor is extremely valid. However, as elaborated further below, these unexplained phenomenons can also be drawn to a plethora of other possible interpretations; from our ability to not detect the presence of other bodies or the shift in the fabric of space time which bends

the light significantly, altering the actual orbit significantly, the list goes on and on. Yet if we decide to leave this as in the form of dark matter, its level of contributions is significantly overlooked.

Introduction

The research question that I had finalized for the purposes of investigation is, “To what extent does the presence of Dark matter influence the movement of celestial bodies?”. The research question is perhance saturated with significant room for ambiguity, especially upon interpretation of findings presented. This paper will solely focus however on the effect of dark matter again subjective to personal interpretation and moreover its influence on celestial bodies. The investigation will tackle, again, various scenarios of different bodies along with interpreting it from a myriad of mathematical equations again that define the influence of dark matter in a plethora of different ways.

Dark matter as a whole is invisible to the naked eye. However, it demonstrates properties that are similar to regular matter, such as establishing and thus drawing in objects due to its gravitational field. Yet as stated to Heisenberg's uncertainty principle, we can not know the position and movement of a quantum particle at the same time, the same correlation can be drawn on the basis of dark matter, and that is perhaps why it contributes significantly towards the unexplainable in the universe.

In detail, the significance of truly comprehending the organic nature of dark matter and moreover its presence on movements of celestial bodies is perhaps one of the most significant questions to answer, that if accomplished, can leave a deep mark in the fields of astrophysics especially when investigating after on on records of the past presented to us. Moreover, these findings can be extended upon other fields, especially in quantum mechanics such as in the behavior of photons and its correlation to the fabric of space time, mathematical equations, especially in the fields of vector calculus and multivariable derivation and integration and moreover classical newtonian, lagrangian, hamiltonian and einstein based physic models that we interpret our world in.

Overall, the research provides a significant scope of merit and moreover contribution towards a plethora of fields. Additionally, it has been phrased to cover its influence in specific to celestial bodies for the purposes of the investigation outlined post literature review.

Literature review

1. Introduction

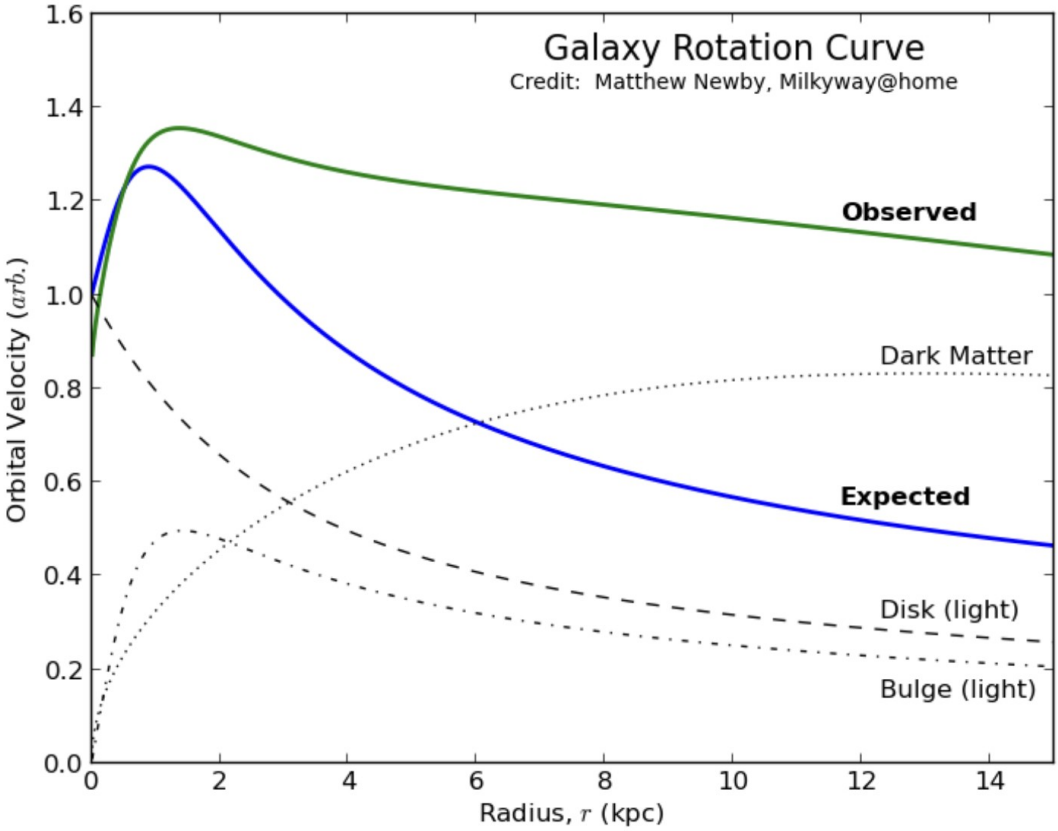
Dark matter as a whole compromises up to 27% of the Universe's mass-energy content. Additionally, it is vital for explaining a plethora of phenomena that occur within a galactic scale as a whole, such as but not limited to, flat rotation curves and gravitational lensing. While its presence, especially upon a galactic scale where its effects are significantly more amplified, when observing its influence down to distances below light years, and within solar systems become distorted and left to a significant degree of debate. This review will analyze and infer from theoretical models, observational studies and a plethora of proposed measurement techniques relevant to the influence of dark matter on celestial bodies.

2.Theoretical Perspectives and Models

2.1 Galactic Versus Local Influence

Dark matter has well been established as the cause behind a plethora of anomalous rotation curves of galaxies. When we examine the seminal work by Vera Rubin for example, we can infer that stars that orbit towards the edges of galaxies orbit at roughly the exact same speeds as those found near the center, usually containing a supermassive black hole, which locks all of the celestial matter in orbit. Such observations can imply that dark matter behaves as in the form of a diffused halo which alters the dynamics of expected gravitational behaviours of objects on such scales.

However, when we again examine this upon a scale of solar systems, the predicted density of dark matter is extremely low. Astronomy Magazine (n.d.) for instance explains such a contradiction, as so far, the calculated mass of dark matter, within our solar system alone is practically negligible and in ways out of proportion to its expected mass. With such an argument, its easy to argue that dark matter as a whole has no influence on the scale of the solar system. Yet this still does not explain effects such as flat rotation curves and thus presents itself with a plethora of more questions than answers.



The figure above outlines the rotational speed of a plethora of galaxy planetary bodies and the speed of which they revolve. Looking at every galaxy including the milky way, post a distance of 1-2 Kiloparsec, there appears to be little to no anomalies and significant variations in the speed of objects and how they revolve. What does stand out however is how planetary bodies appear to contradict logic; as they appear closer to the center (0.5KPC-2 KPC) , they appear to be significantly slower.

2.2 Modeling Dark Matter's Local Effects

Iorio (2010), had provided a theoretical framework which was intended to examine how dark matter influences the movement of planetary bodies. This had been achieved by tweaking aspects of Newtonian Mechanics not through formulas but through set values, Iorio had concluded that a perfectly spherical and symmetric dark matter distribution would contain secular perturbations in the orbital elements of planets and satellites. However, these

contradict even in the most optimistic of simulations, measurements of real life instances and do not explain anomaly cases such as the unexplained acceleration of Pioneer 10 and 11 towards the Sun and deviations in the movement of Saturn's moons as observed by the Cassini Probe.
Dark matter can also be used to explain the behaviour of the universe as a whole. Simulations like the Bolshoi and Illustris projects demonstrate extremely well that dark matter is responsible for the chronology in the formation of galaxies and clusters whilst also determining the behaviour of celestial bodies even from light years apart, despite the disorientation formed by gravitational lensing, also thus proving the significance of it on a local scale.

3. Observational Strategies and Measurement Techniques

Historically, dark matter was simply analyzed by comparing results of what was observed in trajectory and velocity to what had been expected at a given moment using available context. However, there have been attempts of finding more effective ways that also are more sensitive to detecting the presence of dark matter as a whole. Landau (2022), for example discusses such methods whilst considering the general assumptions of dark matter as a whole.

One of which is utilizing spacecraft and analyzing the expected movement of the spacecraft to its actual movement per minute. This would allow the most minute of tugs which indicate the presence of dark matter even at such a miniscule scale. Additionally, one other such method includes utilizing a reflective ball from a significant distance in which the suns gravitational field weakens; observations from such could help map out differential measurement of dark matter influence.

These methods would be crucial, especially for examining their influence on planetary bodies within the range of a solar system and help draw and conclude more accurate results, whilst aiming to help explain phenomena such as the flat rotational curve.

We investigate upon previous attempts towards investigating and comprehending the nature

and form of dark matter as a whole, the room for interpretation is significant amongst the scientific community. However, there are a plethora of concepts that the general consensus has come to agree upon.
So far, these include dark matter being completely invisible when perceived from any form of electromagnetic radiation, that it has its own gravitational field and thus must be able to influence the higgs boson field as well. After this, the level of interpretation is so limited, such research is based on scientific assumption over concrete facts as a whole.
Keeping in mind the limited scope of understanding we have entirely on the nature of dark matter, it is important to keep an open perspective and consider observations through every single perspective possible.

4. Conclusion

After analyzing various sources, we can infer that dark matter is perhaps an extremely misunderstood concept. Its not a concrete state of matter, rather invisible; it doesn't occupy space or volume but exhibits the properties as if so it had been there, which in turn could help for mapping out its presence. Yet in the conundrum of various planetary bodies and objects, its presence becomes rather impossible to determine its concrete position even if the universe had 3 bodies present moving at once.

Dark matter as a whole though does present itself with a plethora of influence on celestial bodies; especially from a galactic scale. Yet even within the range of solar system, its effects can be felt even if so not traced as a whole. The question now is to what extent are they influential. Do they contribute towards destroying the fabrics of classical Newtonian, Lagrangian, Hamiltonian and quantum physics along with the concepts of general relativity or just produce anomalies?

Methods

1. PhET Colorado gravity and orbits simulations.

Experiment/Simulation setup

-Utilizing the solar system simulation on PhET Colorado, I was able to simulate the presence of dark matter, by incorporating small masses within the simulation.
-These masses did get affected by the movement of regular matter, yet were set to an initial

velocity of 0 km/h to ensure more accurate results.
-Various trials were conducted with the presence of 1 and even 2 bodies of dark matter.

Data collection

-The independent variable is the position and mass of each dark matter body. The orbit in particular is controlled along with all of the relative masses.
-What we are measuring is the trajectory followed by the orbiting object which is also the dependent variable.

Analysis technique

-Comparing the shifts in position and velocity at various stages are the main indicators for identifying the extent of dark matter's influence.
-One other indicator would be the final distance after 1 complete orbit.

Software used

The PhET Colorado simulation will be used to simulate the entire orbit of the objects at a 2D scale.

Hypothesis

-The presence of dark matter bodies will be significant enough to cause a collision between the center mass and the orbiting body, but never move the centering mass significantly or cause the orbiting body to fall out of orbit.

2. W.I.M.P (weakly interacting massive particles)

Experimental/Simulation setup

-A simulated germanium (Ge) crystal lattice is used to model WIMP interactions
-Monte Carlo simulations along with molecular dynamic techniques are incorporated to observe nuclear reactions.
-The cross section lattice is based upon the standard assumptions of dark matter models.

Data collection

-The dependent variable being measured is the recoil energy of the impacted atoms found within the simulation crystal.
-The threshold for Ge-Ge bonds has been implemented to determine the threshold.
-The recoil energy output is then plotted against the atomic index for comparison.

Analysis and technique

-Energy distributions are analyzed in a statistical manner to identify trends and peak events.
-The comparison with the theoretical displacement are crucial for identifying the likelihood of

an atomic displacement.
-The results can thus be extrapolated for larger scale phenomena as such recoils can easily be found in the presence of dark matter to the extent of influencing the movement of celestial bodies.

Software and tools

-Python along with libraries such as, NumPy, SciPy, Matplotlib
-Simulations softwares like Geant4 or custom Monte Carlo simulations, to better model the interactions.

Hypothesis

-The recoil energy post collision will reflect a possible shift in energy and thus influence on orbit, but will never exceed a threshold of 80% as its on the verge of an impossible occurrence due to the strength of such bonds relative to the theoretical impact of the collision.

3. Gravitational lensing simulation

Experimental/Simulation Setup

A simulated Navarro-Frenk-White (NFW) profile is used to model the dark matter density distribution, ideal for demonstrations of N-Body simulations over large spaces.
Light bending due to the gravitational potential is calculated using the lens equation detailed below.
The simulation has been designed to measure the deflection angle and magnification across different radial distances from the halo's center, and thus reflect the influence of dark matter on the position of objects.

Data Collection

The dependent variable being measured is the deflection angle of the light emitted from the object and magnification of the light after set distances.
The distance from the center of the halo acts as the independent variable.
The deflection angle and magnification are plotted as a function of radial distance from the halo center and are set constant..

Analysis and Technique

The deflection angle is computed using the lens equation manipulated by the Jacobian set for this vector field and lens equation:

$$\alpha(r) = \frac{4GM(r)}{c^2 r}$$

The magnification is determined by the Jacobian of the lens equation.
Results are analyzed to determine the effect of dark matter halo mass on the light path and lensing strength.

Software and Tools

Python programming will be used along with libraries such as NumPy, SciPy, Matplotlib to help conclude comprehensive test.
Computational tools for numerical integration and matrix inversion to solve the lens equation.

Hypothesis

The gravitational lensing would be enough to distort the position of the objects but never their final magnitudes as light always stays the same speed in a vacuum and only the direction of the light is being altered.

4. N-Body Simulation of Galactic Dynamics

Experimental/Simulation Setup

A simulated Navarro-Frenk-White (NFW) profile is used to model the gravitational potential of the dark matter halo but instead of the 3rd method, multiple bodies are present without any form of gravitational lensing.
N bodies are initialized with random initial positions and velocities within the halo during each simulation.
The simulation computes gravitational forces and updates the bodies' positions and velocities over time using the Verlet Integration method. Unlike the former method, thai directly measures the objects post the presence of external forces which vary per simulation.

Data Collection

The dependent variable being measured is the orbital trajectory and velocity of the bodies over

time per simulation.
The initial positions and velocities act as independent variables yet are set to random.
The orbital trajectories and velocity curves are plotted against radial distance from the halo center and are always constant unlike the former method..

Analysis and Technique
The gravitational force is computed using:

$$\alpha(r) = \frac{4GM(<r)}{c^2 r}$$

The motion is integrated using the Verlet Integration method:

$$v(r) = \sqrt{\frac{GM(<r)}{r}}$$

The velocity distribution is analyzed to determine the dark matter halo's influence on the orbital structure.

Software and Tools
Python along with libraries such as NumPy, SciPy, Matplotlib.
Simulations performed using custom N-body solvers for gravitational interactions and observations will thus be recorded from there.
Hypothesis
The gravitational pull of the halo would be able to influence the direction more than its speed (not velocity) as the halo would just shift the direction and even if so, ever so slightly under such specified circumstances.

Methodology

The experiment will be conducted using a plethora of simulations and methods. Elaborated on below are the selected methods of observing the influence of dark matter on celestial bodies. One of the key perspectives that I would like to elaborate on before I continue with elaborating on the methodology is the behavior of dark matter that I consider in the experiments. Dark matter is treated as such as an invisible body of mass that forms a gravitational field. Considering the limitations of such simulations, I had simulated the influence by placing bodies of matter with mass between 2 orbiting bodies and thus inferred on the influence of the body as a substitute for dark matter. Which also leads to another crucial perspective; dark matter can be influenced in its position and in such magnitude as does normal matter. Though dark matter is still subjective to interpretation on its nature in terms of its existence as a matter that registered under the higgs boson, its nature can as of now be deduced as invisible mass.

1) PhET Colorado Simulations

There are a select few simulations within PhET Colorado, that allows us to investigate the influence of dark matter on celestial bodies. Though this only gives us a 2 dimensional perspective on the influence, it can help us elaborate and moreover help establish the fundamental basis of a plethora of trends whilst also contributing towards inferences that we can make in the process.

An interesting observation that I had deduced during the experiment is rather regarding the orbit of the moon when its path is mapped out on a cartesian plane, with the sun as the origin. The observation is with regards to the crest and the trough of the sort of flower shape that is formed in its orbit.

From the figure shown to the left hand screen, we can observe the path of the moon in the purple line, which forms a sort of flower figure. Considering that no variables were changed during its completion of an orbit, it completes 12 wavecycles.

The interesting observation, which we can deduce, is the relationship between the distance between the crests and troughs of the wave cycle. The distance, being 324 pixels approximately between crests and 200 between troughs yields the value of ϕ or 1.618 when we divide the displacements between the crests and troughs.

2. WIMP Detection

The study investigates the interactions of Weakly Interacting Massive Particles (WIMPs) with a crystalline germanium (Ge) lattice, analyzing recoil energy distributions and their potential implications for dark matter research.

Experimental Setup

Crystal Lattice Model: A simulated germanium (Ge) lattice was created, consisting of a fixed number of atoms (100) which were implemented to observe the impact of WIMP-induced energy transfer.

WIMP Interaction Simulation:

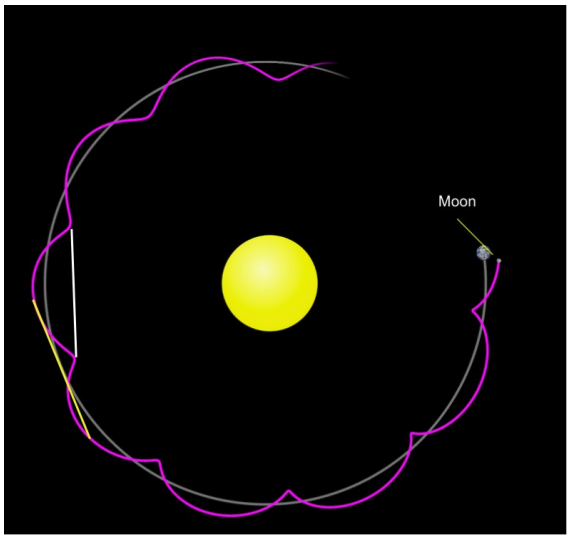
WIMPs in this experiment were assumed to possess non-relativistic velocities (~220 km/s, typical of dark matter in the Milky Way halo).

Their mass was varied randomly between 10 GeV/c^2 and 100 GeV/c^2 which was implemented to analyze different energy distributions.

Elastic scattering events had been modeled using a standard dark matter-nucleus interaction cross-section.

Recoil Energy Calculation:

The energy transferred to each atom was calculated using the equation:



where:

Er = Recoil energy of the nucleus (in GeV)

mNm_NmN = Mass of the nucleus (in GeV/c^2)

mWm_WmW = Mass of the WIMP (in GeV/c^2)

vvv = Velocity of the WIMP (in km/s)

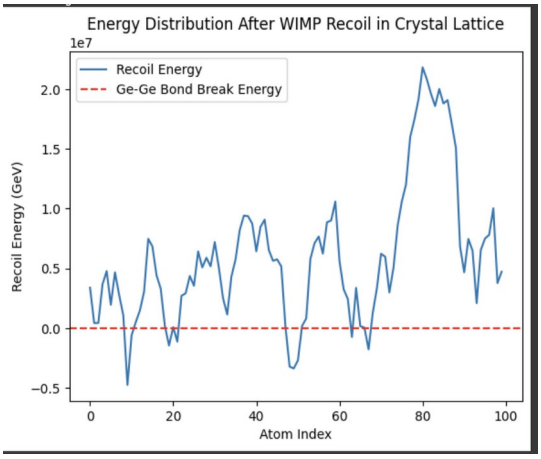
θ\thetaθ = Scattering angle (in radians)

Bond Breakage Threshold: The energy required to break apart Ge-Ge bonds had been set and indicated by the threshold (showcased by the red dashed line in the graphs), determining the probability of lattice disruption.

Data Collection & Analysis

A Monte Carlo approach was used to simulate numerous WIMP interactions across the lattice.
Recoil energy was then plotted against an atomic index to visualize energy distribution and its relation to the threshold.
The fraction of atoms exceeding the bond break energy was calculated to estimate the potential structural disruption within the lattice.

Results



The simulation showed a wide range of recoil energies, with some atoms experiencing energy levels significantly exceeding the bond break threshold, and others showcasing minimum breakage.
The percentage of bonds at risk was **91.00%**, indicating a high probability of structural disruption.
A peak recoil energy of **~2 × 10^7 GeV** was observed, suggesting that heavier WIMPs transfer more momentum to the atoms.
The variation in energy distribution aligns with expectations from elastic WIMP-nucleus scattering models.
These findings support the feasibility of using crystal lattice disruptions as indirect evidence for dark matter interactions.
This methodology and results section forms the basis for understanding how WIMP interactions could be detected through crystal damage, ultimately correlating to the broader astrophysical implications of dark matter's influence on celestial orbits.

3. Gravitational Lensing

The study simulates the bending of light due to gravitational lensing caused by a dark matter

halo, using the Navarro-Frenk-White (NFW) profile to model the mass distribution and calculate deflection angles and magnifications.

Simulation Setup

NFW Profile Model:

A Navarro-Frenk-White (NFW) profile was defined to represent the density distribution of the dark matter halo:

$$\rho(r)=\rho0rrs(1+rrs)2\rho(r) = \frac{\rho_0}{\frac{r}{r_s}(1 + \frac{r}{r_s})^2}\rho(r)=rsr(1+rsr)2ρ0$$

Where:
rsr_srs = scale radius

ρ0\rho_0ρ0 = characteristic density

Light Deflection Simulation:

- The deflection angle was computed using the lens equation:

$$\theta-\beta=4GM(r)c2r\theta - \beta = \frac{4GM(r)}{c^2 r}\theta−β=c2r4GM(r)$$

where:

- θ\thetaθ = observed angle
- β\betaβ = actual position of the source
- M(r)M(r)M(r) = mass enclosed within the radius

Magnification Calculation:

- The magnification factor was computed as:

$$\mu=1det(J)\mu = \frac{1}{\text{det}(J)}\mu=det(J)1$$

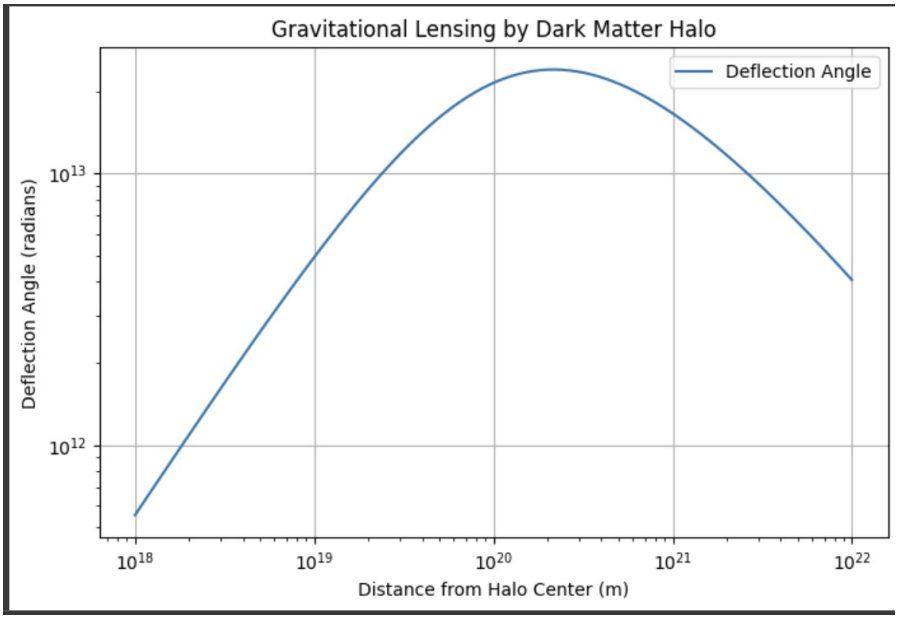
where JJJ is the Jacobian of the lens equation.

Data Collection & Analysis

Deflection angles and magnification were plotted as a function of the radial distance from the halo center.

Results were compared with theoretical predictions to validate the accuracy of the simulation. Trends in magnification and deflection patterns were analyzed to assess the influence of dark matter distribution on gravitational lensing.

Results



4. N-Body Simulation of Galactic Dynamics

The study simulates the gravitational interactions of N bodies within a galactic dark matter halo, using the Verlet integration method to calculate motion and energy distribution over time.

Simulation Setup

Initial Conditions:

A fixed number of particles (N = 1000) were placed randomly within a spherical volume representing the halo.

Initial velocities were assigned based on a Maxwell-Boltzmann distribution consistent with the halo's gravitational potential.

Gravitational Force Calculation:

- The gravitational force between two bodies was calculated using:

$$F = \frac{G m_1 m_2}{r^2}$$

where:

- m_1, m_2 = masses of the two bodies
- r = distance between the bodies
- G = gravitational constant

Verlet Integration:

- Positions and velocities were updated using the Verlet integration method:

$$\vec{r}(t + \Delta t) = 2\vec{r}(t) - \vec{r}(t - \Delta t) + \frac{\vec{F}(t)}{m} \Delta t^2$$

Data Collection & Analysis

- The trajectory and velocity of each body were tracked over time.
- Orbital dynamics and velocity dispersion were analyzed to identify gravitational clustering and halo stability.
- Results were compared to theoretical predictions to validate the model's accuracy.

Results

Discussion

With using PhET Colorado for simulating dark matter along with the other 3 methods of testing, here are the various interpretations on examining the extent of dark matter influence on the orbit of celestial bodies.

Perfect candidate for explaining unexpected orbits or planets leaving orbits

In history there have been many instances where bodies have seemingly gone off track, whilst others which should have didn't. Or why in such cases elements forget the laws of chemistry momentarily and form functional groups and imbalance sigma and pi bonds out of nowhere. Dark matter as a whole is perhaps a perfect candidate; its sheer concentration within the universe, coupled with its symmetry helps contribute significantly towards alterations not only in orbit direction but trajectory.

Dark matter influences the movement of celestial bodies producing anomalies yet sticking to the laws of physics.

This is perhaps the best conclusion on answering exactly to what extent does dark matter influence the orbits of celestial bodies. As through not only from a plethora of literature reviews but also through simulations, there are of course extreme anomalies produced in terms of results of orbits, yet dark matter as a whole is not directly violating any laws of physics or chemistry as a whole.

Dark matter is just a force as one could say. It propagates itself confining to the laws of physics, but not like other fluid based behaviours and laws such as thermodynamics.

Dark matter is just a flaw in the spacetime continuum and higgs boson field

In chemistry, we have learned that matter, especially fluids as dark matter is deemed to behave such, can occupy the space and the form of which space they are in. Yet if so, why doesn't dark matter reach a state of equilibrium in the space time continuum? Or why doesn't dark matter reduce to its simplest form.

In recent years, we have examined various new forms of matter, theoretical and concrete from strange matter and virtual particles to more theoretical based elements such as tachyons. Yet in chemistry and quantum chemistry along with fermat's least action principle, everything obtains a state of rest or aims to achieve that. This is the reason for hawking radiation, the reason for the decay of protons and neutrons to elementary particles, the decay of more complex functional groups and thus elements.

Yet what if dark matter is simply obstructions in the higgs boson field? Such fluctuations would allow for a theoretical space of "matter" to be generated, causing bends in the space time

continuum. This could also be caused by particles that cause such as those found in Femtochemistry. This would be a perfect candidate for explaining not only why such trends of symmetry are present such as the speed of objects in galaxies orbiting the center are equal, but also confine to current understanding.

Dark matter does not exist

This is a diverse perspective that I had in mind prior towards writing the paper, yet had only seemed to expand in nature as in when I had conducted the experiment. The perspective is that dark matter simply does not exist.

With such a simple statement come a significant degree of perspectives each with their own merits and depth. One of which is simply the effects of the gravitational fields of other bodies on the light waves that we perceive. Considering the fabric of spacetime is massive; most observations are dozens of not thousands or millions of light years away, its extremely probable that the presence of various overlapping gravitational fields, which end up warping the fabric of spacetime causing phenomena such as gravitational redshift or otherwise altering the position and velocity from our perspective of the object.

With this in mind, another alternate possibility is simply the presence of matter that has not been detected or simply put, the light waves that would help us understand the true nature of unnatural observations have not either reached us or have been scattered so much that they are impossible to be detected in similar wavelengths or even at all. Its sort of like looking at a room through a keyhole; no matter how much you peek, you wont be able to see and thus explain everything which is going on.

This perspective is perhaps one of the most overlooked perspectives of the influence of dark matter, yet if proven true, would completely remove the concept of dark matter, dark energy and other related concepts as well.

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